# Greedy Method

## APS 2 (SPIS 2016)

Algorithmic Problem Solving
Wednesday, August 10

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# General Problem solving

- In general, when you try to solve a problem, you are trying to find a solution from among a large space of possibilities. You usually do this by making a series of decisions and continuing based on what the new state of the problem has become.

# General Problem Solving

- In general, when you try to solve a problem, you are trying to find a solution from among a large space of possibilities. You usually do this by making a series of decisions and continuing based on what the new state of the problem has become.

- If you have no information about which choice is best, you may have to use exhaustive enumeration to try out all possibilities.

- This might take a long time to do. What are some other ideas in general?

# Backtracking

- Instead of blindly passing through all possibilities, you may be able to keep picking choices until you reach a dead-end and backtrack.
- Backtracking may lead to many possibilities as well.
- If you Backtrack in a very clever way, you may skip whole branches of possibilities. (This is the topic for next week.)

# The Greedy Method

- In some cases (not all!!!!!), there is sufficient structure that allows you to reach the correct solution by just picking the straightforward "best" decision.

- This is called the Greedy Method.

# Cookies

- Suppose you are the cookie monster and you have a 6x6 sheet of freshly baked cookies in front of you. The cookies are all chocolate chip cookies but they may have different sizes
- If you are only allowed to take six cookies.
- Devise an algorithm to ~~do this~~.

choose the biggest combination of cookies.

# Cookies

| 56 | 76 | 69 | 60 | 75 | 51 |
|---|---|---|---|---|---|
| 61 | 77 | 74 | 72 | 80 | 58 |
| 82 | 97 | 94 | 88 | 99 | 92 |
| 47 | 68 | 59 | 52 | 65 | 40 |
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99
97
94
92
88
85

1. What is an algorithm you could use to select the *best* option?

(The best option means that the sum of all the cookie's sizes is the highest possible.)

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94

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99+97+94+92+88+85=555 grams.

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76+80+99+68+85+73=481

82+97...=552

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404

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99+81+72+69+47+46=414

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92+78+75+73+72+68=458!!!!!!!!

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# The Greedy Method

- As you have seen, the Greedy Method does not always work. Because of this, in order to use the Greedy Method, we must prove the correctness of the algorithm.

# The Greedy Method (WARNING!!)

- As you have seen, the Greedy Method does not always work. Because of this, in order to use the Greedy Method, we must prove the correctness of the algorithm.

- Furthermore, for a single problem, there may be more than one potential greedy strategy i.e. more than one way to choose the "best" possible choice at each step. The problem may be solved in one way but not the other.

# The Greedy Method (disproving)

- To show that the greedy method does not work, you must present a "counter-example"
- In other words, a counter-example is an example where using the greedy method does not give you the best possible solution.
- Sometimes counter-examples are easy to find but sometimes they can be more challenging.

# The Greedy Method (don't lose hope)

- If the Greedy method doesn't work. (if you have presented a counter-example), there still may be a way to solve the problem without exhaustive enumeration.

# The Greedy Method (don't lose hope)

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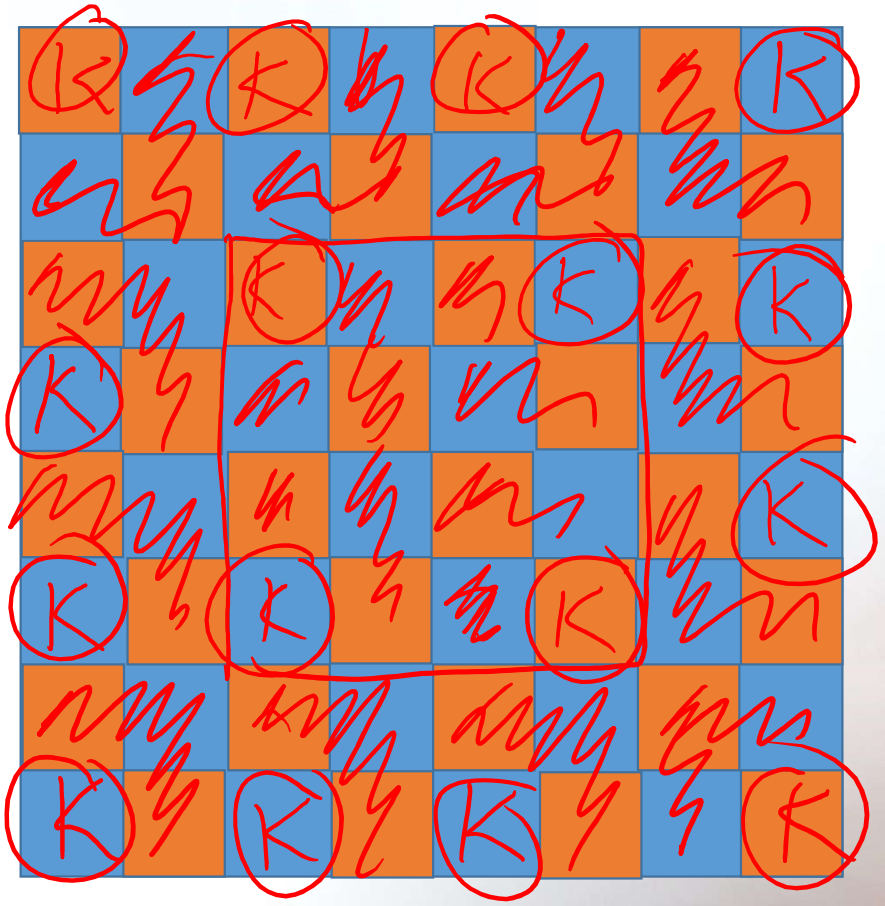
Any ideas on how to solve this problem in general?

# The Greedy Method (Homework)

For the homework this week, it is important to

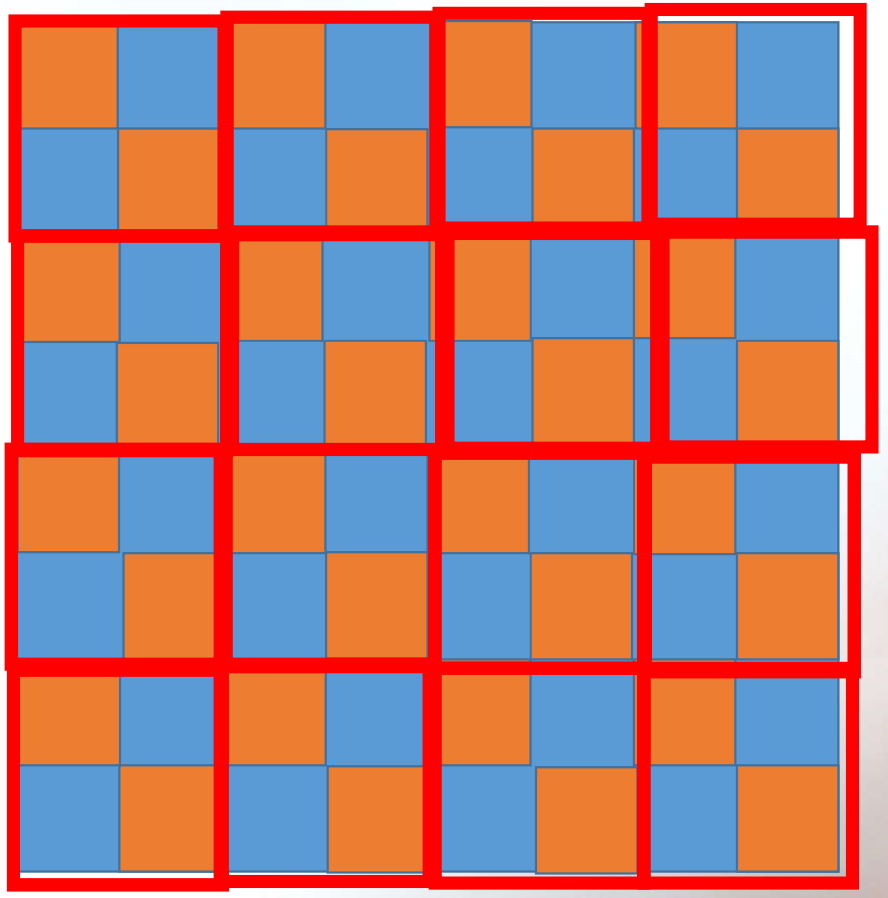
- Clearly describe your algorithm
- Discuss (prove) why your algorithm is correct
- (Or present a counter-example and try again.)

# Greedy method (Example1)



- Place the greatest possible number of kings on a 8x8 chessboard so that no two kings are placed such that they are “attacking”

# Greedy method (Example1 proof)



- There can be no more than 16.
- We have a solution for 16
- Therefore 16 is the best possible solution.

# The Greedy Method (Example2)

You have the following chores to complete:

- Walk the dog (15 minutes)
- Mow the lawn (60 minutes)
- ~~Shovel the snow~~ (45 minutes) Cleaning your room
- Take out the trash (2 minutes)
- Clean the pool (45 minutes)
- Wash the windows (75 minutes)
- Wash the car (30 minutes)
- ~~Cook dinner~~ (20 minutes) Homework.

You would like to complete as many jobs as you can in say 3 hours. Clearly describe an algorithm to do this. Discuss its correctness

# The Greedy Method (Example3)

You have the following chores to complete and each pays:

- Walk the dog (15 minutes, $5)
- Mow the lawn (60 minutes, $50)
- Shovel the snow (45 minutes, $20)
- Take out the trash (2 minutes, $2)
- Clean the pool (45 minutes, $15)
- Wash the windows (75 minutes, $60)
- Wash the car (30 minutes, $15)
- Cook dinner (20 minutes, $5)

You would like to earn the most money possible in 2 hours. Clearly describe an algorithm to do this. Discuss its correctness.

# The Greedy Method (Example4)

Suppose each one of you has a rumor. You can only share the rumor by passing a note.

The goal is to share the rumors so that everybody in the class knows all the rumors in the least possible number of note passes.

(you may write several rumors that you know in one note.)

Describe your algorithm and why it works.